Electrical and Electronics Engineering
2024-2025
Master Semester 2

Course
Smart grids technologies

DFT-based Synchrophasor Estimation Algorithms – DFT and Aliasing

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Outline

- Synchrophasor estimation algorithms
- Signal sampling
  - Aliasing and Nyquist-Shannon theorem
- The Discrete Fourier Transform (DFT)

Synchrophasor estimation algorithm

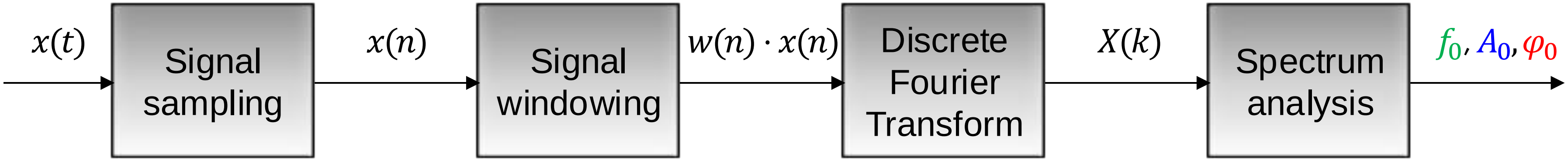
Challenges and requirements

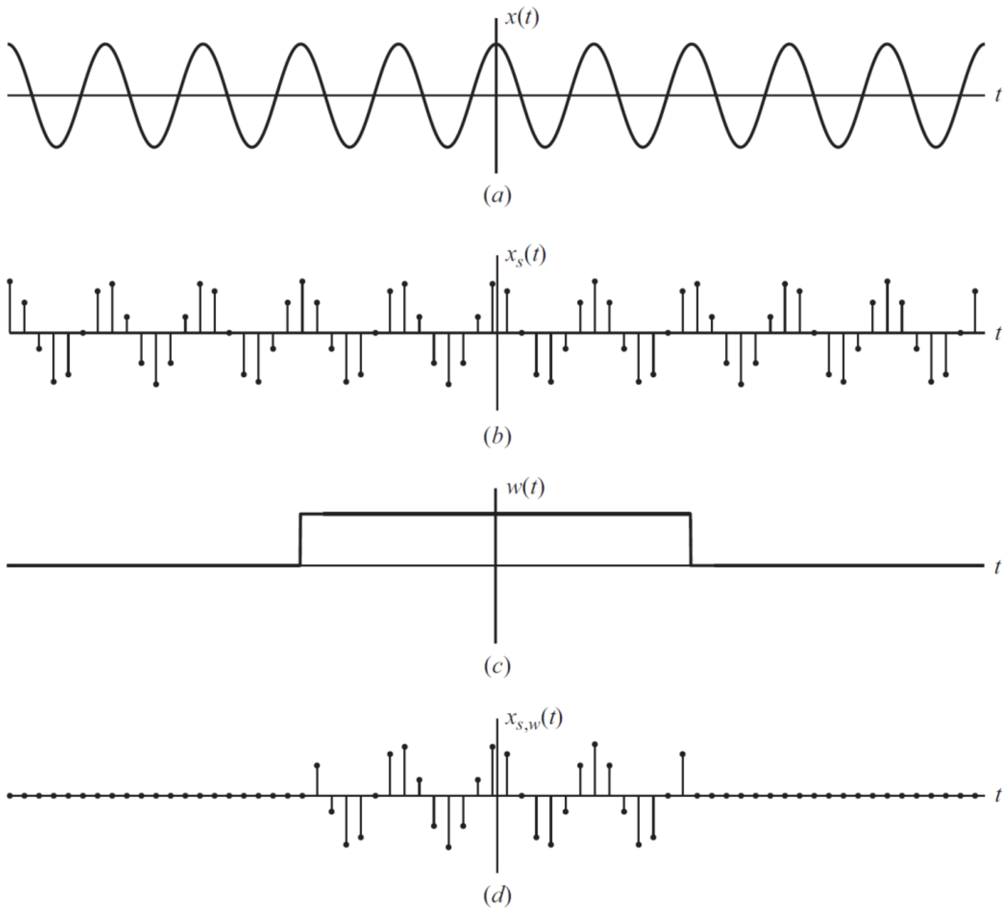
- Estimation of the main tone of a signal whose frequency is not known a-priori
- Static phasor representation vs. dynamic behavior of electrical systems
- Algorithm's accuracy vs. algorithm's response times
- Algorithm's performances (50 estimations per second) vs. algorithm's computational complexity

Class	Typical algorithms	Advantages	Drawbacks
DFT based	Fourier analysis (e.g., [1])	Low computational complexity, harmonic rejection	Spectral leakage, Harmonic interference
	Interpolated DFT (e.g., [2])		
Wavelet based	Recursive wavelet (e.g., [3])	Harmonic rejection	Computational complexity
Optimization based	WLS (e.g., [4])	Accurate when used in combination with other methods	Non deterministic: driven by optimality criteria
	Kalman Filter (e.g., [5])		
Taylor series based	Dynamic Phasor (e.g., [6])	Intrinsically dynamic	Computational complexity

DFT-based synchrophasor estimation algorithms

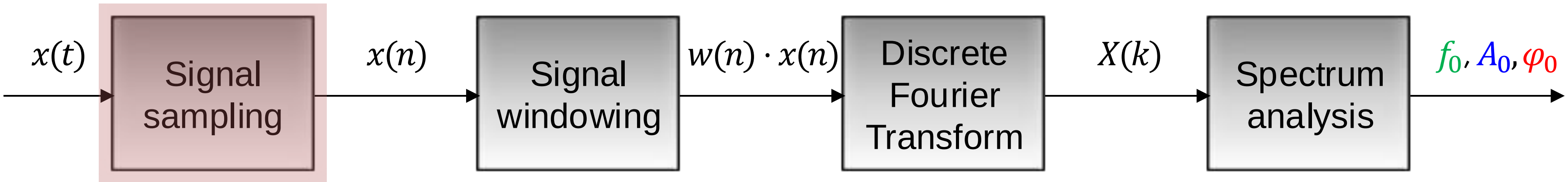
Measurement chain





Signal sampling

Basics

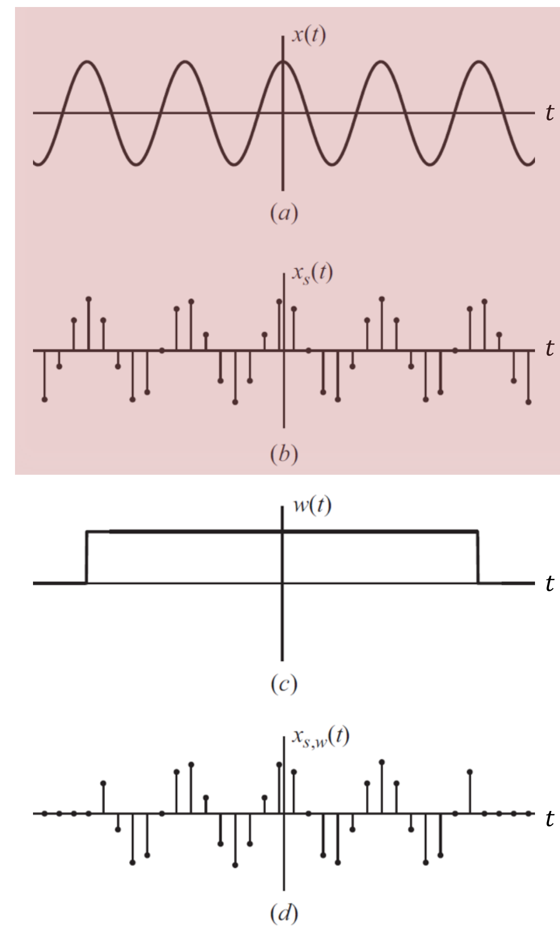


The waveforms of a generic power system (node voltages and/or branch/nodal currents) are analog signals characterized by distortion. In order to implement signal processing algorithms into a digital systems, we need first to **convert the analog signal to its digital representation.**

As seen in the lecture #1, the sampling process can be modeled as a periodic impulse train modulated by the amplitude levels of the signal:

$$x_S(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_S) = \sum_{n=-\infty}^{\infty} x(nT_S) \cdot \delta(t - nT_S)$$

being δ the Dirac delta function and $T_S = 1/F_S$ the sampling time and F_S the sampling frequency



Signal sampling

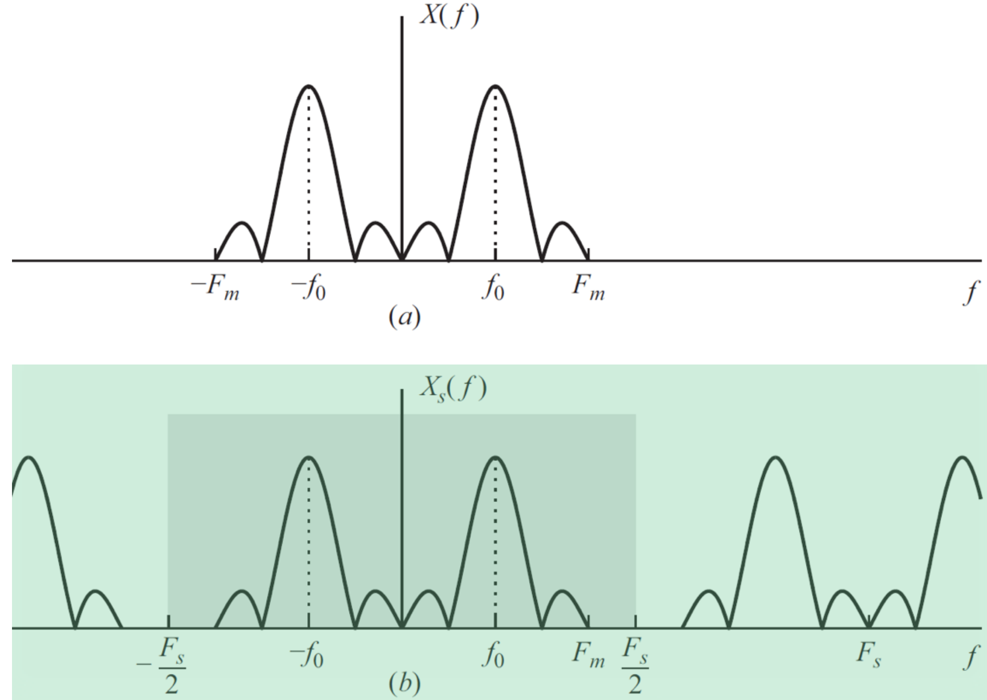
Spectrum of the sampled signal

$x(t) \quad x_S(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$

↓ ↓

$X(f) \quad X_S(f) = X(f) * \frac{1}{T_S} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_S}\right)$

$$= \frac{1}{T_S} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T_S}\right)$$



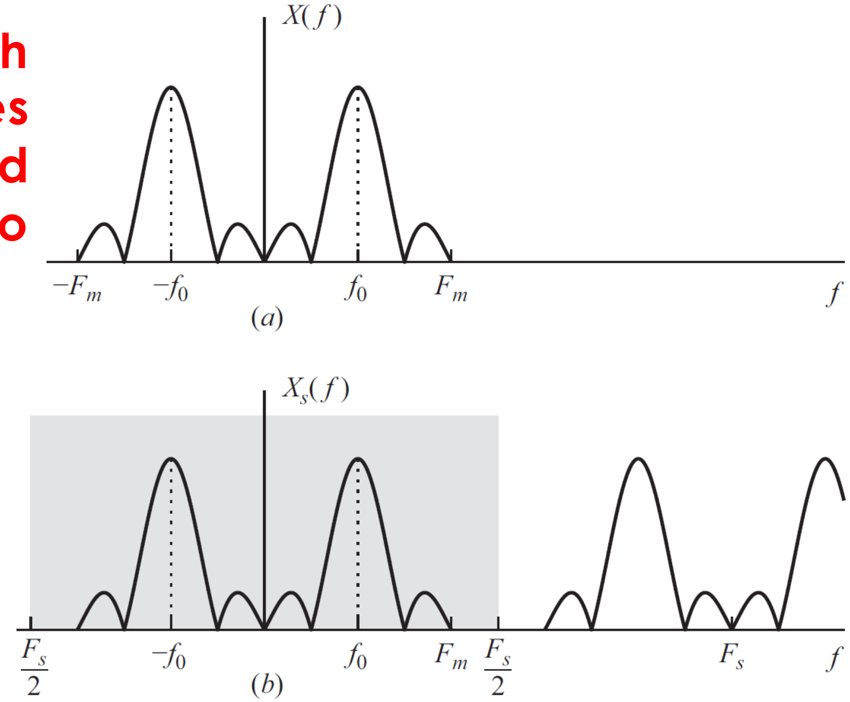
The spectrum of the sampled signal contains **copies** of the original spectrum $X(f)$ centered at integer multiples of the sampling frequency F_S.

If the signal is band-limited with bandwidth $F_m < F_S/2$, the spectrum copies are not overlapped. Therefore from $X_S(f)$ it is possible reconstruct $X(f)$ (i.e., $x(t)$) by low-pass filtering the base-band copy.

Signal sampling

Aliasing

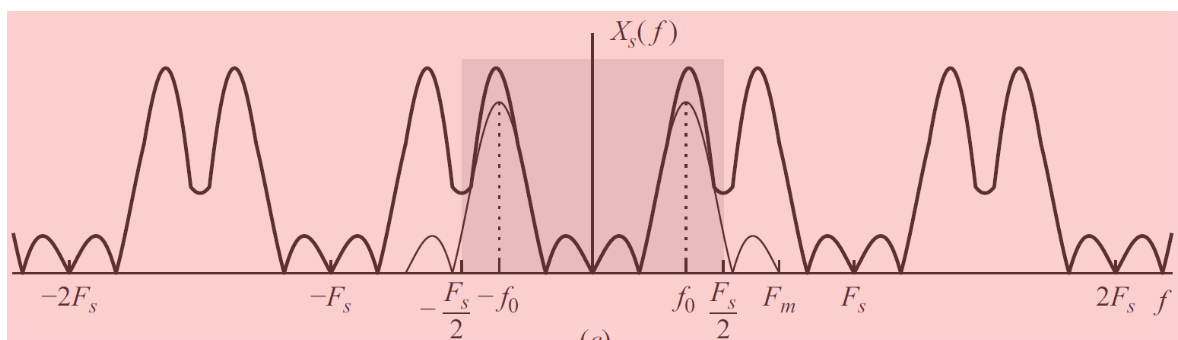
If the signal is NOT band-limited with bandwidth $F_m < F_S/2$, the spectrum copies are overlapping so that when they add together, the original spectrum $X(f)$ is no longer recoverable by low-pass filtering.



This phenomenon is usually referred as **aliasing** and does not allow to recover $X(f)$ from $X_S(f)$.

Aliasing is usually corrected by two possible approaches:

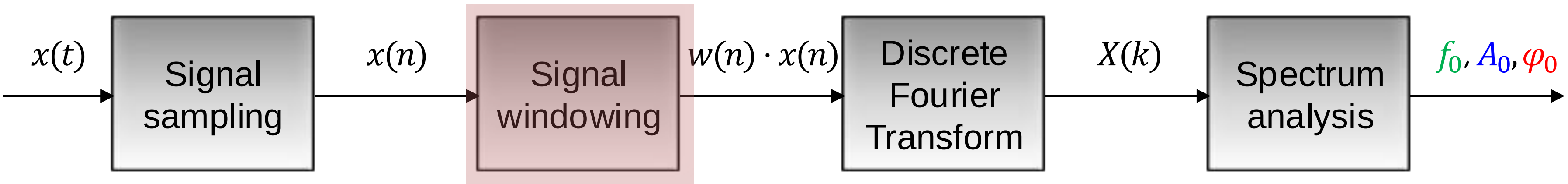
1. Using anti-aliasing filters that limits the signal bandwidth (reduced accuracy)
2. Increasing the sampling frequency to values much larger than the highest spectrum component contained in the sampled signal.



$F_m < F_S/2$ ➔ *(Nyquist-Shannon sampling theorem)*

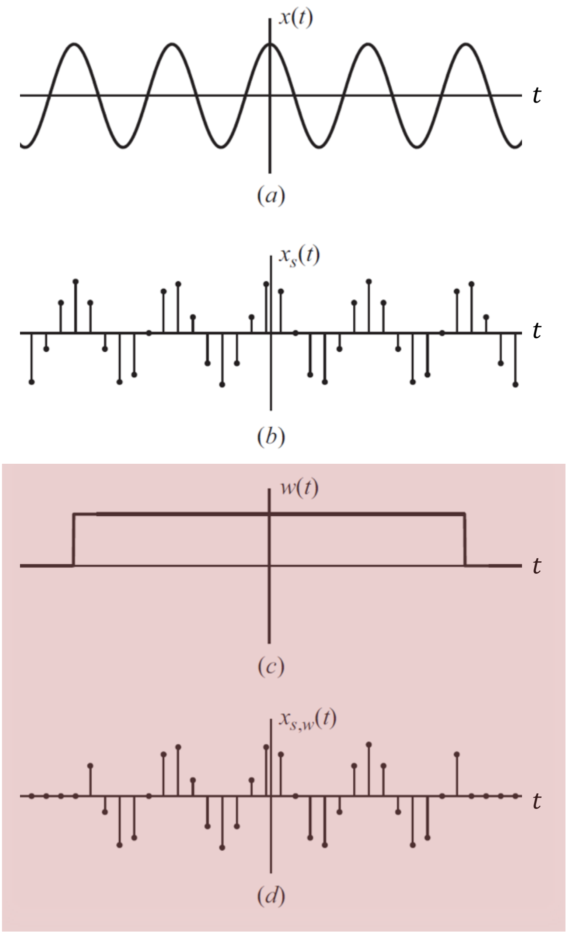
Signal windowing

Basics



In order to be able to perform spectral analysis on the acquired signal, the signal need to be sectioned in portions, also called **windows**.

The primary purpose of the window is to limit the time-extent of the signal so that the signal can be considered stationary within the duration of the window (i.e. the more rapidly the signal changes the shorter the window should be)



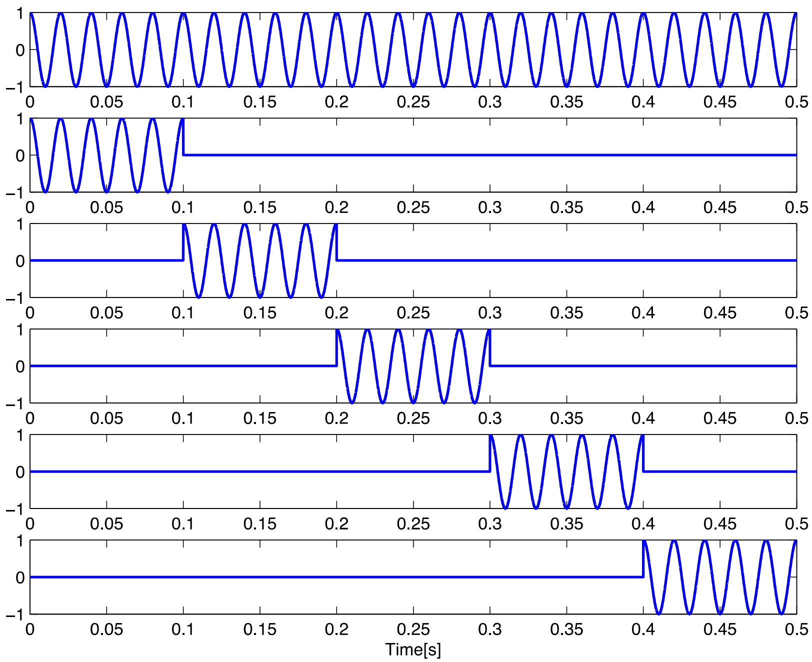
Signal windowing

Window parameters

- **The window length** T: it has to be selected as a trade-off between frequency resolution and time resolution.

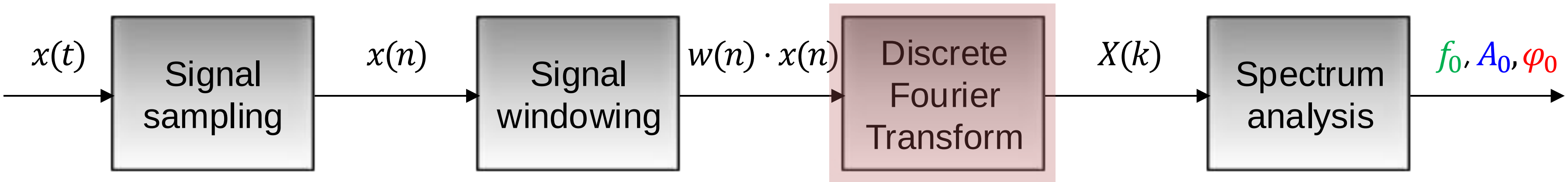
- **The window profile** $w(t)$: a naive approach, the **rectangular window**, involves simply truncating the dataset before and after the window, while not modifying the contents of the window at all. However, as we will see, this is a poor method of windowing and causes **spectral leakage**.

$$w_R(t) = \begin{cases} 1 & \text{if} - T/2 \leq t \leq +T/2 \\ 0 & \text{else} \end{cases}$$



The Discrete Fourier Transform (DFT)

Basics



A different interpretation of the DFT

For a **generic rectangular window**, the **single DFT bin** can be computed using the following compact relation:

$$X(k) \triangleq \frac{2}{B} \sum_{n=0}^{N-1} w(n)x(n)W_N^{kn}, 0 \le k \le N-1$$

being $x(n)$ the sampled signal, $w(n)$ the discrete window function and B the **normalization factor** (the use of this factor **ensures that the amplitude of the spectrum components is directly comparable with the original time domain signal**):

$$B \triangleq \sum_{n=0}^{N-1} w(n)$$

The so-called **twiddle factor** is:

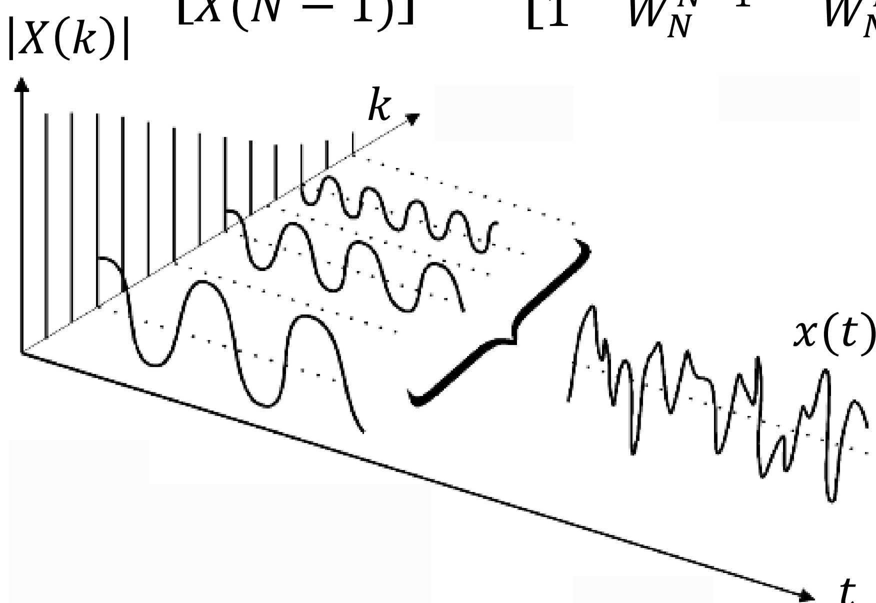
$$W_N = e^{-j2\pi/N} = \cos(2\pi/N) - j\sin(2\pi/N), W_N^k = e^{-j2\pi k/N}$$

The Discrete Fourier Transform (DFT)

A more intuitive interpretation

$$X(k) \triangleq \frac{2}{B}\sum_{n=0}^{N-1} w(n)x(n)W_N^{kn}$$ → The DFT can be equivalently written in **matrix form** for a more intuitive understanding of its logic

$$\begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(k) \\ \vdots \\ X(N-1) \end{bmatrix} = \frac{2}{B}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & W_N^{(k)_N} & W_N^{(2k)_N} & \cdots & W_N^{((N-1)k)_N} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & W_N^{N-1} & W_N^{N-2} & \cdots & W_N \end{bmatrix}\begin{bmatrix} w(0)x(0) \\ w(1)x(1) \\ \vdots \\ w(k)x(k) \\ \vdots \\ w(N-1)x(N-1) \end{bmatrix}$$



It can be shown that the columns of matrix $\left[W_N^{(k)_N}\right]$ are **linearly independent**. Therefore, they are a base of $\mathbb{C}^N$. In this respect, the DFT provides, in the frequency domain, the **projections of the windowed signal on that base of $\mathbb{C}^N$**.

Note that $w(k) = 1$ for a rect. window.

The Discrete Fourier Transform (DFT)

- The DFT is a numerical tool that plays a central role in the implementation of a variety of digital signal-processing algorithms.

- It is the equivalent of the continuous Fourier Transform for **finite sequences of data characterized by length N and separated by sample time T_S.**

- The DFT can be correctly interpreted as **frequency-discretized version of the continuous-time Fourier transform**. The DFT samples are called **bins** and are **equally spaced by the frequency interval** $\Delta f = 1/T$ that is called **frequency resolution**.
  - The phase of each DFT bin specifies the relative alignment of the input signal to each complex exponential
  - The magnitude of each DFT bin is proportional to the power contents of each complex exponential

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